THE SAT 1500+ PLAYBOOK

Advanced Tactics for the Hardest Questions



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Chapter 2: The Hardest Math Archetypes

The hardest Math questions are rarely hard because they use advanced content. They are hard because they hide familiar content behind unfamiliar structure, chain two or three ideas together, or ask for something slightly different from what you naturally compute. This chapter covers eight archetypes of hard Math questions. For each one you get the rule, a worked example, the common trap, and a quick check.

Archetype 1: Disguised Quadratics

Many equations that do not look quadratic become quadratic after one substitution. The signal is an expression that appears twice, once "squared" and once plain: x^4 and x^2, 4^x and 2^x, x and sqrt(x), or a whole parenthetical expression such as (x - 5) appearing squared and unsquared.

> **Key rule:** When you see an expression and its square in the same equation, name the expression u, solve the quadratic in u, then translate every u back into x. The question is almost always about x, not u.

Worked example. What is the sum of all solutions to $4^x - 6(2^x) + 8 = 0$?

Notice that $4^x = (2^x)^2$. Let $u = 2^x$. The equation becomes $u^2 - 6u + 8 = 0$, which factors as $(u - 2)(u - 4) = 0$, so $u = 2$ or $u = 4$. Translate back: $2^x = 2$ gives $x = 1$, and $2^x = 4$ gives $x = 2$. The sum of the solutions is 3.

> **Trap:** Reporting 6, the sum of the u-values. The question asks about x. This is the Halfway Answer trap from Chapter 5: a correct intermediate value that is not what the question asked for.

Two other families of disguised quadratics show up often. Rational equations become quadratics after you multiply by the common denominator, and radical equations become quadratics after you square both sides. Both processes can create solutions that do not satisfy the original equation. A rational equation cannot use a value that makes a denominator zero, and squaring can introduce a value at which the square root would have to equal a negative number. Always check candidates in the original equation, or let Desmos do the checking (Chapter 6).

Quick check. What is the product of all solutions to x - 7sqrt(x) + 12 = 0?

Answer: Let u = sqrt(x). Then $u^2 - 7u + 12 = 0$, so u = 3 or u = 4, and x = 9 or x = 16. Both check ($9 - 21 + 12 = 0$ and $16 - 28 + 12 = 0$). The product is 144. If you answered 12, you multiplied the u-values.

Archetype 2: Structure Instead of Solving

Hard quadratic questions often give you information about the solutions without giving you the solutions. The fastest route uses the structure of $ax^2 + bx + c = 0$:

- The sum of the solutions is -b/a.
- The product of the solutions is c/a.
- The x-coordinate of the vertex is -b/(2a), which is also the average of the two solutions.
- The discriminant $b^2 - 4ac$ tells you the number of real solutions: positive means two, zero means one, negative means none.

None of these formulas is on the Bluebook reference sheet, and neither is the quadratic formula, so you need them memorized.

Worked example. The solutions to $2x^2 - 14x + c = 0$ are p and p + 3, where c is a constant. What is the value of c?

The sum of the solutions is -(-14)/2 = 7. So p + (p + 3) = 7, which gives p = 2, and the solutions are 2 and 5. Their product is 10, and the product must equal c/2, so c = 20. Check: $2x^2 - 14x + 20 = 2(x - 2)(x - 5)$.

> **Trap:** Setting the product equal to c instead of c/a. That gives c = 10. Whenever the leading coefficient is not 1, divide by it before using sum and product.

Quick check. In the equation $kx^2 - 6x + 1 = 0$, k is a constant. The equation has no real solutions. Which of the following could be the value of k: 4, 8, 9, or 12?

Answer: No real solutions means $b^2 - 4ac < 0$, so $36 - 4k < 0$ and k > 9. Only 12 works. The value 9 gives exactly one solution, not zero.

Archetype 3: Parameter Questions

A parameter question includes a constant such as a, b, or k and asks what value of the constant produces a certain result. They are hard because they test whether you understand what solutions mean, not just whether you can find them.

Here are the conditions you need:

- Two expressions are equivalent (“for all values of x”) exactly when their corresponding coefficients match after expanding.

Answer Explanations: Set A

1. **B.** A linear system has no solution when the x- and y-coefficients are proportional but the constants are not. The x-coefficients have ratio 3/6 = 1/2, so the y-coefficients need -k/4 = 1/2, which gives k = -2. Check the constants: with k = -2, the first equation is 3x + 2y = 7, and doubling it gives 6x + 4y = 14, not 5, so the lines are parallel and distinct. Choice C drops the negative sign on -k (Sign Flip). Choices A and D come from inverting the ratio (6/3 = -k/4 gives -8, and 6/3 = k/4 gives 8); with those values the lines intersect at exactly one point.
2. **22.** Let $u = x^2 - 3$. Then $u^2 - 5u - 6 = 0$, so $(u - 6)(u + 1) = 0$ and $u = 6$ or $u = -1$. That means $x^2 = 9$ or $x^2 = 2$, so the real solutions are 3, -3, sqrt(2), and -sqrt(2). Their squares are 9, 9, 2, and 2, which sum to 22. A common error is entering 11 (counting only positive solutions, the Lost Solution trap) or 5 (the sum of the u-values, the Halfway Answer).
3. **B.** Exactly one solution means the discriminant is zero: $k^2 - 4(2)(18) = 0$, so $k^2 = 144$ and $k = 12$ (k is positive). The single solution is $-k/(2a) = -12/4 = -3$. Check: $2x^2 + 12x + 18 = 2(x + 3)^2$. Choice A uses -k/2, forgetting to divide by 2a (Halfway Answer from an incomplete formula). Choice C is the Sign Flip. Choice D combines both errors.
4. **28.** Multiply factors: $1.25(1 - p/100) = 0.90$. Divide: $1 - p/100 = 0.72$, so $p = 28$. Entering 35 (adding 25 and 10) or 15 (subtracting) is the Wrong Whole trap: the decrease is a percent of the increased price, not of the original.
5. **C.** Because DE is parallel to BC, triangle ADE is similar to triangle ABC with length ratio AD/AB = 6/10 = 3/5. Areas scale by the square of that ratio, 9/25, so the area of ABC is $27 \times 25/9 = 75$. The quadrilateral is the rest: 75 - 27 = 48. Choice D is the Halfway Answer (the area of ABC). Choice B uses the length ratio for area ($27 \times 10/6 = 45$), the Unit and Scale Slip, and choice A subtracts 27 from that wrong total.
6. **A.** Factor the right denominator: $x^2 - x - 2 = (x - 2)(x + 1)$. Multiply both sides by it: $x(x + 1) + 4(x - 2) = 6$, which simplifies to $x^2 + 5x - 14 = 0$, or $(x + 7)(x - 2) = 0$. The value $x = 2$ makes denominators zero, so it is extraneous. The only solution is -7, and the sum is -7. Choice B is -b/a for the quadratic, which includes the extraneous root (Lost or Extra Solution). Choice C is the extraneous root alone. Choice D is the Sign Flip of the answer.
7. **D.** The given line, $3x - 4y = 8$, has slope 3/4, so line ℓ has slope -4/3. Using the point (2, -1): $y + 1 = (-4/3)(x - 2)$, so $y = (-4/3)x + 8/3 - 1 = (-4/3)x + 5/3$. Choice A uses $(x + 2)$ instead of $(x - 2)$, a Sign Flip. Choice B uses the parallel slope 3/4. Choice C uses -3/4, the negative of the slope instead of the negative reciprocal.